Optimising the javelin throw in the presence of prevailing winds

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Abstract

Javelin flight is strictly governed by the laws of aerodynamics but there remain well-entrenched but misleading views about the factors involved. This paper gives a simplified but fundamentally accurate view of the physics and dynamics of javelin flight and describes a freely downloadable software implementation of these. The model has been calibrated against two sets of published competition data and predicts distances within about 1%. Finally, it goes on to show how throwing parameters can be optimised for given prevailing wind force and direction.

1 Overview

There have been numerous studies of the biomechanics of javelin throwing in the literature such as those by [1], [6] and [7] but there still appear to be many misunderstandings in practice about the actual aerodynamics of throwing in general and javelin throwing in particular in spite of particularly comprehensive treatments such as that by [2]. Other notable attempts to predict the distance a javelin flies from its release parameters include the work of [5] who use a neural net and they report accuracies of better than 5% and usually half this using this technique.

Of particular interest here is the effect of the prevailing wind on the javelin throw which is significant but has not been treated in such detail primarily because of the complete lack of controlled experiments in which it was measured. Given that a failure to understand the prevailing conditions and their impact on the throw can make all the difference between success and failure, this paper will attempt to address this deficiency. Many academic studies do not result in accessible technology so this paper will address this also. The physics will be described first followed by a mathematical model and finally by a description of a freely available calibrated modelling software package which embodies these principles, [4].

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The software will only model post-1986 javelins. The essential difference was that the centre of gravity was moved forward in 1986 because of the prodigious distances being achieved by such throwers as Uwe Hohn, ($>$ 104m.). The only other alternative would have been to move the javelin outside the main arena which would have deprived onlookers of seeing one of the most spectacular of all field events. There have been other minor changes also but the effect of this re-balancing dominates and is essentially two-fold:-

- The average distance flown is reduced by around 10%
- Moving the centre of gravity forward means that it is now about 6cm in front of the *centre of pressure*. The centre of pressure is defined to be the point at which the aerodynamic forces of lift and drag on the javelin apply. This means that there is an upward lifting force 6cm aft of the downward force acting through the centre of gravity which is situated close to the front edge of the handle. The effect is that the javelin experiences a turning moment in the vertical plane which forces the point down and which therefore causes it to stick in removing any ambiguity as to where it has first landed. The women's 600gm javelin was similarly adjusted in 1999. Unfortunately, the disastrous current performance of many 700 gm javelins is because no such steps have been taken with this javelin with the result that the manufacturers seem to ignore it presumably to promote greater distances. Unfortunately, the flight characteristics would suggest that the centre of gravity and centre of pressure are effectively co-located meaning that it will very frequently land flat even at its rated distance and the author has personally witnessed flat landings at 60m with most modern manufacturers in the ESAA (English Schools Athletics Association) championships with this most frustrating implement as well as half of all throws in the British Master's championships being flat landers, (male U17, and master's M50 and M55 use the 700gm.)

  Note that this is something of a simplification because the centre of pressure moves during the flight as the javelin changes its angle to the air-flow over it although it does not move very much for a thin aerofoil such as the javelin, provided the angle of attack, (the angle between the javelin and the air-flow across it), remains small, [3]. This has been studied in detail in wind tunnel experiments using the javelins of the day by [2]. Amongst many other things, they show plots of the movement of the centre of pressure against the actual angle of attack, (discussed below), which confirms this. For actual angles of attack of less than 10 degrees, the movement is no more than around 5cm for any particular javelin although the actual position can vary considerably from javelin to javelin.

2 Some basic principles

The javelin is not so much a throw as a long pull. This is a most important lesson to impart to young athletes as they gradually begin to pick the event up. By far the most significant factor is the speed of the javelin as it leaves the hand, assuming it is aligned properly. Furthermore, the distance it travels

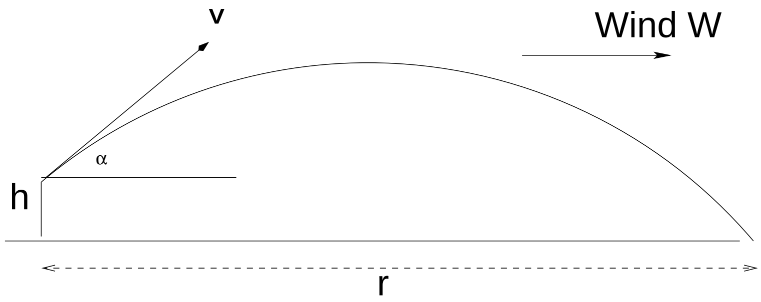

Figure 1: The nomenclature used in the accompanying text. A javelin is launched with velocity V at angle α from a height h m. above the ground with a wind of W m/s and flies a distance of r m.

is proportional to the square of the speed so that an increase of 10% in the speed increases the distance by a factor of 1.1 x 1.1 = 1.21 or 21%. Of course increasing speed by 10% is harder than it sounds and is completely dependent on the biomechanical properties of the thrower.

So how is the javelin accelerated up to its final speed before leaving the hand ? In essence, the hand pulls on the javelin for a distance of some 2m in adult champion throwers accelerating the javelin from around 6 metres per second relative to the ground (allowing for the run-up) to around 30 metres per second relative to the ground. Note that Newton's law determines how successful this is as it is not simply the force which is applied. The musculature has to accelerate not only the javelin but also the athlete's arm and upper body. Newton's law states that the acceleration achieved is the force the athlete can apply divided by the mass which is being accelerated. If the athlete has a heavy musculature, the force which can be applied has to overcome the inertia of this mass also to achieve the maximum acceleration. This is why good javelin throwers are usually long and rangy. With respect to the muscles used in the upper arm, the bicep is simply added mass which hardly contributes as the bulk of the acceleration in this phase of the throw is applied by the tricep, so big biceps equal wasted mass.

3 The dynamics of the throw

Figure 1 shows the nomenclature used.

3.1 Simple mathematical treatment

Let t_u be the time taken to reach the highest point of the flight from leaving the hand. Let t_d be the time taken to reach the ground from the highest point of the flight. Let s_u be the height reached above the point of release and let g be the acceleration due to gravity. Let h be the height at which the javelin is released. Let r be the distance covered by the javelin.

Then, resolving upward, the time taken to reach maximum height resisted by gravity is given by

$$0 = V sin\alpha - gt_u \tag{1}$$

Knowing V, α and g then gives t_u. The maximum height reached above the hand is given by:-

$$0 = V^2 sin^2\alpha - 2gs_u \tag{2}$$

The time taken to reach the ground from maximum height is given by

$$s_u + h = 0 + \frac{1}{2}gt_d^2 \tag{3}$$

Knowing h, s_u and g then gives t_d. The total time the javelin is in the air is therefore $t_u + t_d$. The horizontal distance covered by the javelin is therefore given by:-

$$r = V sin\alpha(t_d + t_u) \tag{4}$$

This gives the classic parabolic shape of an ideal body in flight and is a useful approximation to the flight of a javelin. As will be seen later, this is only accurate to within about 5% even for well-delivered javelins and to do it more accurately and account properly for wind and all the other factors, the aerodynamics must be re-introduced.

3.2 Enhanced mathematical treatment including aerodynamics

Drag, lift and pitch Linearised theory for airflow around an object like a javelin ([10]) allows us to approximate the drag force when the javelin is parallel to the wind as that for a thin Joukowski airfoil

$$D = 2\gamma\pi\rho V^2\epsilon^2 \tag{5}$$

where ϵ is the radius of the head of the javelin, ρ is the density of air and γ is a constant. There is no lift in this case. Pitch (or attack) where the javelin makes a small angle with the wind can be simulated by increasing this value to represent more of the javelin's aspect being presented to the air-flow around it. A simple calculation reveals that the increased area presented assuming the javelin is cylindrical with radius ϵ (not a bad assumption given that its tapered at both ends and thicker in the middle) is given by

$$\pi\epsilon^2 + 4L\epsilon sin(\delta) \tag{6}$$

where L is the length of the javelin and δ is the angle of attack. The total longitudinal drag is therefore given approximately by

$$F_D \approx 2\gamma\rho V^2(\pi\epsilon^2 + 4L\epsilon sin(\delta)) \tag{7}$$

For moderate values of α, the angle of attack also gives a perpendicular lift, [3] of

$$F_L \approx 4\pi\rho V^2\alpha \tag{8}$$

These terms need to be incorporated into the governing equations of motion.

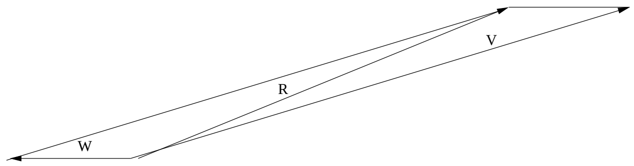

Figure 2: The parallelogram of velocities when a javelin is launched with velocity V at angle α to the ground into a wind of W m/s. The resultant velocity is R

More on attack Attack is rather more subtle than it appears. In aerodynamics, the angle of attack or *actual* angle of attack as it will be referred to here, is the angle between the long axis of the javelin and the air-flow past the javelin. In previous studies such as that by [1], the angle of attack is measured as the angle between the long axis of the javelin and the direction of movement of the centre of gravity relative to the ground using a high speed camera. It is important to realise however that this is only the *apparent* angle of attack rather than the actual angle of attack. Previous authors have certainly been very well aware of this, [1], [2], but as will be seen here the difference is crucial when trying to account for prevailing wind conditions.

Consider 'throwing through the point' into the presence of a head wind as shown in Figure 2. The effect of the head wind is to reduce the velocity of the javelin but also *to change the direction of its centre of gravity.* There is therefore a negative apparent attack angle relative to the ground even though the javelin was thrown through the point and relative to the javelin, the air-flow is still parallel to the long axis of the javelin so the actual angle of attack is still zero. Only when the prevailing wind is zero are the apparent angle of attack and the actual angle of attack the same. With non-zero winds they are not the same and it is perfectly possible for example, for the javelin to have a negative apparent angle of attack as filmed from the ground and a positive actual angle of attack generating lift (and also greater drag). It is of course the actual angle of attack which generates the aerodynamic forces of lift and additional drag.

Finally it should be noted that although a zero actual angle of attack generates the least drag because the air-flow is parallel to the long axis of the javelin, it is not obvious that this condition at launch would cause the javelin to fly the furthest as it also generates less initial lift. Pre-1986 javelins used to be thrown deliberately with a high actual attack angle to achieve greater distance, [8]. Analysis of a number of post 1986 champion throwers however often shows the javelin launched with a slight apparent down-angle of a few degrees whatever the wind direction. The brief discussion above suggests that inferring the actual angle of attack may not be so simple and one of the points of this paper is to investigate the implications of this and to clarify strategies for predicting these effects to find the combination which leads to the longest throw given the prevailing wind conditions and biomechanical properties of the thrower. The resultant velocity R and the apparent angle of attack can be found using the cosine and sine rules.

Javelin types To complicate things further, javelins come in basically three types, Headwind, Tailwind and neither (General), although not all manufacturers use precisely these terms. Javelin manufacturers are allowed very little flexibility in the position of the centre of gravity and it is normally checked at weigh-in. The only significant remaining parameter they have some control over is then the centre of pressure. In the mid-80s one attempt to influence this was made by producing roughened tails which move the centre of pressure closer to the centre of gravity in such a way as to simulate javelins similar to the pre-1986 types. Since this undermined the whole purpose of the 1986 re-modelling, it was quickly banned. Nowadays only modest changes can be made which are related to the nature of the prevailing wind. To summarise, the three kinds of javelin have the following properties:-

- Headwind. Pointed javelins which it is commonly believed fly a little further into a headwind but will not fly as well as a tailwind javelin into a tailwind.
- Tailwind. Blunter nosed javelins to attempt to break the boundary layer flow around the javelin a little earlier with the effect of bringing the centre of pressure a little closer to the centre of mass giving a smaller pitching moment to keep the nose up a little longer. The common belief is that these function better in a tail wind.
- General. These javelins are usually pointed and are not optimised in any particular way.

In the author's experience as latterly a coach and formerly a thrower, folklore about the behaviour of these javelins abounds but the mathematical model which follows does not support the folklore which the author found puzzling for some time. Certainly headwind and general javelins have less drag because the value of ϵ is smaller but the centre of pressure also moves so there is a counterbalancing effect. This was finally cleared up by [8] in a personal communication describing recent comments about Headwind and Tailwind javelins made by one of the pioneers of modern javelin design, Dick Held. In essence, Held made it clear that the javelins were originally distinguished only by a blunter nose - the shafts were identical. Apparently nobody would throw the blunt javelin because it would obviously increase the drag, however *Held knew that the blunt javelin outperformed the sharp javelin in almost every environment*, (for very similar reasons to the higher performance of the rough-tailed javelin but not as emphatic). In order to get people to use it, he called it a Tailwind and people then began to use it in tail winds and so the belief grew.

Full equations of motion This section simply turns the above physical factors into mathematical form so that they can be used to predict the flight of a javelin. This development assumes the javelin is delivered with no yaw or lateral actual angle of attack, (i.e. as seen from above) but this is easy to add and in practice did not make much difference to the quality of the model's data-fitting.

Let m be the mass of the javelin, δ be the attack angle above the angle of delivery and let the position vector of the javelin relative to the delivery line be (r,s,q) where r is in the direction of the javelin flight, s is upwards and q is across from right to left. Resolving in each of these three directions using Newton's law (mass x acceleration = Force) and including the lift and drag terms in equations 8 and 7, leads to the following coupled non-linear differential equations:-

$$m\frac{d^2s}{dt^2} = -mg + 4\pi\rho\beta_j\alpha((\frac{dr}{dt})^2 + (\frac{ds}{dt})^2 + (\frac{dq}{dt})^2) \tag{9}$$

$$m\frac{d^2r}{dt^2} = -2\gamma_j\rho(\pi\epsilon^2 + 4L\epsilon sin(\delta))((\frac{dr}{dt})^2 + (\frac{ds}{dt})^2 + (\frac{dq}{dt})^2) \tag{10}$$

$$m\frac{d^2q}{dt^2} = 0 \tag{11}$$

Here β_j and γ_j are fitting constants *which depend on the javelin type essentially through the point radius ϵ and partly through the shape.* These equations are integrated forward in time using a Kutta-Merson fourth-order procedure with control over global error, (see for example, [9]), to give the position vector (r,s,q) at all times after launch. The integration method used does not really matter much as the resulting errors are very much less than other sources of error although [2] did suggest that a fourth-order procedure at least should be used which almost any procedure worth its salt is capable of achieving or exceeding. If ϕ is introduced as the azimuthal angle of delivery with 0 corresponding to the centre of the arc, then the initial conditions at t = 0 are:-

$$r = 0 \tag{12}$$

$$\frac{dr}{dt} = Vcos(\alpha)cos(\phi) \tag{13}$$

$$s = h \tag{14}$$

$$\frac{ds}{dt} = Vsin(\alpha)cos(\phi) \tag{15}$$

$$q = 0 \tag{16}$$

$$\frac{dq}{dt} = W_x + Vcos(\theta)sin(\phi) \tag{17}$$

where W_x is the crosswind.

Pitching moment Finally the pitching moment is included by solving:-

$$r\frac{d\alpha}{dt} = -moment \tag{18}$$

concurrently with the equations above, where the moment is the moment of the lifting and drag forces about the centre of gravity. The movement of the centre of pressure has to be parametrised as it is related to the generally unknown behaviour of the boundary layer around the javelin during flight but this behaviour is quite well understood as has been described already.

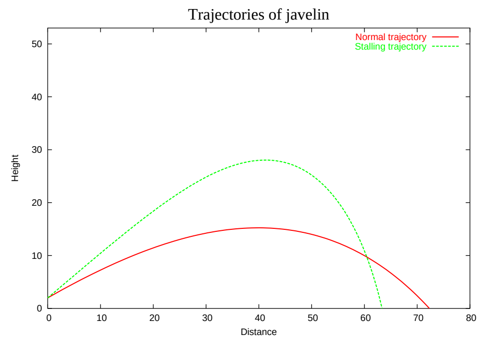

Figure 3: Illustrates the effects of drag on a high trajectory with attack into a head wind.

An example of two trajectories for a Tailwind javelin thrown with the same release velocity into a brisk headwind is shown in Figure 3. The lower trajectory is delivered with no attack and the higher trajectory is delivered with a 5 degree attack angle. The stalling effect can be clearly seen. The qualitative features of javelin flight seem well described by the model and predictions match the observations of [7] well where relevant data was given.

Rotation One factor which was omitted because it appears to be small is the axial rotation of the javelin. For a right hander, coupled with the pitching moment, the javelin will be deflected slightly to the left. It is relatively easy to incorporate this and it will be included at a future stage. The axial rotation has been reported as high as 25 revolutions per second. The reason for the deflection is the gyroscopic principle of precession based on the conservation of angular momentum. There are two rotations, one in the vertical plane when the javelin point pitches downwards and the axial rotation. For a right handed thrower, the javelin precesses to the left if the axial rotation is clockwise from the back. It is important to note however that high axial rotation will therefore resist the downward pitching moment and will intoduce a slight drag and lateral lift effect (yaw). Whether this increases the horizontal distance travelled or not is currently unknown. It may be possible to exploit this by using different grips but this was not considered further here. It is likely however for mechanical reasons and based on the author's personal throwing experience, that the horseshoe grip will lead to slower axial rotation than either of the other two conventional grips.

Stiffness One other factor which was not incorporated is the effect of stiffness of the javelin. If a flexible javelin is mis-hit, a significant amount of the power is expended in increased drag and also in oscillating the javelin, which ultimately dissipates as heat warming the javelin up slightly. The author can recall an extreme example of this from the current world record holder, Jan Zelezny where in one series, the javelin was mis-hit so badly, it appeared as though it would break up in mid-air such was the amplitude of the oscillation. The javelin only travelled about 40m. The next throw from Zelezny was around 90m. For a stiff javelin, there will be a lesser effect. If the javelin is not mis-hit, it doesn't matter. Since this only affects poorer throws, it will not be considered further

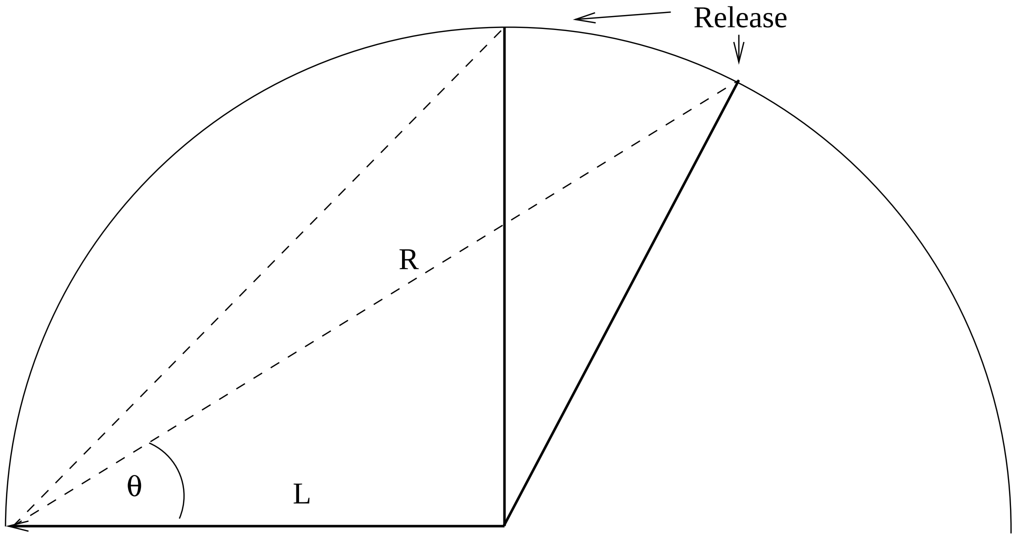

Figure 4: Illustrating the trade-off between increased angle of delivery θ and length of pull R which the javelin thrower can apply. Note that the movement forward of the shoulder pivoting on the base of the back and trunk can be modelled simply by increasing the 'length' of the arm, L.

here although it is very simple to incorporate by using a root mean square aspect ratio for the cross-section as the javelin oscillates.

4 Effects of biomechanical factors

A considerable amount of work has been done on this using for example, the idea of kinetic chains by Bartlett, Morriss and others, [1], [6], [7]. In this analysis, we will consider a very simplified view and will only be interested in a straight trade-off between the angle at which the javelin can be launched and the distance the thrower can hang on to it during the acceleration phase. The simplest representation of this is shown in Figure 4. In this diagram, using the cosine rule,

$$R^2 = 2L^2 - 2L^2cos(180 - 2\theta) = 2L^2(1 + cos(2\theta)) \tag{19}$$

This can be simplified to give

$$R = 2Lcos(\theta) \tag{20}$$

In essence, this states as the delivery angle increases, the range along which the javelin can be pulled gradually diminishes. This is used in the software package itself to optimise the delivery parameters. Its is noted in the paper by [2], page 385 that a linear decrease in release speed with increasing angle of release is observed for the range of release angles found in javelin throwing. It is worthwhile investigating the behaviour of the simplifed model in equation 20. Relative to the body, the relationship between the delivery velocity, arm acceleration and R using equation 20 is given by:-

$$V = \sqrt{(}2aR) = \sqrt{(}4aLcos(\theta)) \tag{21}$$

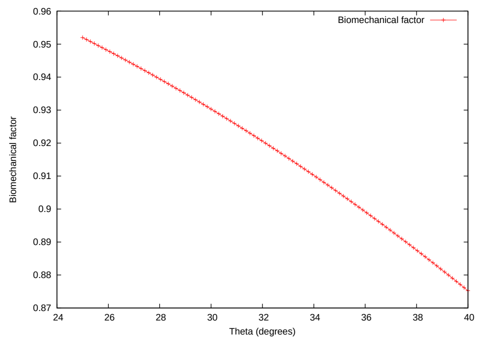

Figure 5: A plot of $\sqrt(4aLcos(\theta))$ against θ for a range of delivery angles typical for javelin throwing where 'a' is the maximum acceleration a thrower can apply and 'L' is the length of the thrower's arm.

Figure 5 shows the behaviour of the right hand side of equation 21 over a typical range of javelin delivery angles. As can be seen, it is very closely linear substantiating the results quoted in [2].

5 Modelling results

The above set of equations handle the qualitative behaviour of a javelin well and are included in the software package *Javelin Flight Analyser*, [4]. A screenshot of the front page is shown as Figure 6. The package is freely available as a self-installing executable for Windows 98/2000/XP from the quoted web location.

5.1 Optimising flight in adverse conditions

Although solving the differential equations used in this model would once have been considered an expensive calculation, modern PCs are sufficiently fast as to be able to do this in a few milliseconds opening up the possibility of searching for optimal solutions in a given set of prevailing wind conditions or for fixed biomechanical factors for a particular athlete. An example of this is shown in Figure 7. Note that the release velocity was fixed in this case. The software package can also allow this to be varied linearly with delivery angle to match the thrower's biomechanics in the manner described above.

6 Calibration

The model has been calibrated against the data recorded in the 1991 World student games as described by [1] and also some anonymous data from the 1993 BAF (British Athletic Federation) championships believed to have been acquired by Bartlett and Morriss but unattributed. These data collectively covered both male and female throwers with distances ranging between 57.22m and 87.42m.

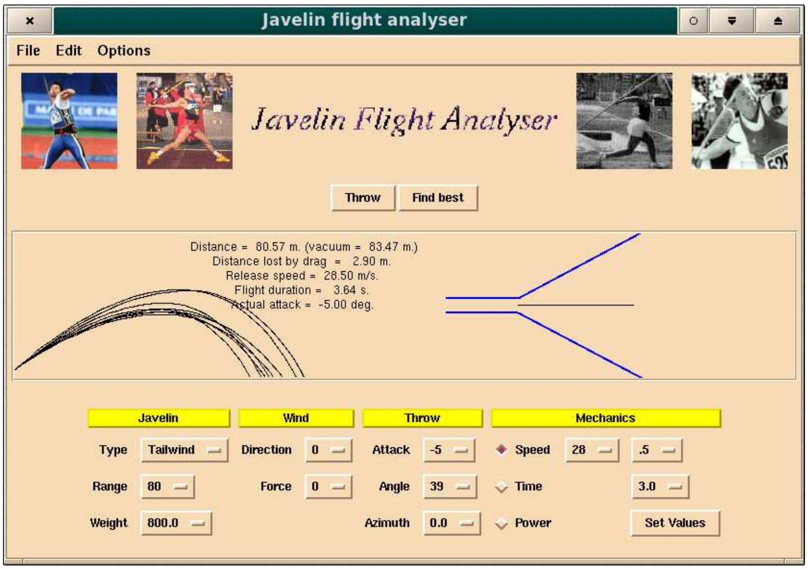

Figure 6: A screenshot of the front page of the Javelin Flight Analyser. The results of simulating the 1993 BAF championships are shown. These covered a range of delivery speeds between 25.4 and 28.5 metres/sec, attack angles of -5 to +7 degrees and delivery angles from 35 to 41 degrees. The very different nature of the trajectories and the non-parabolic effects of air resistance can be clearly seen.

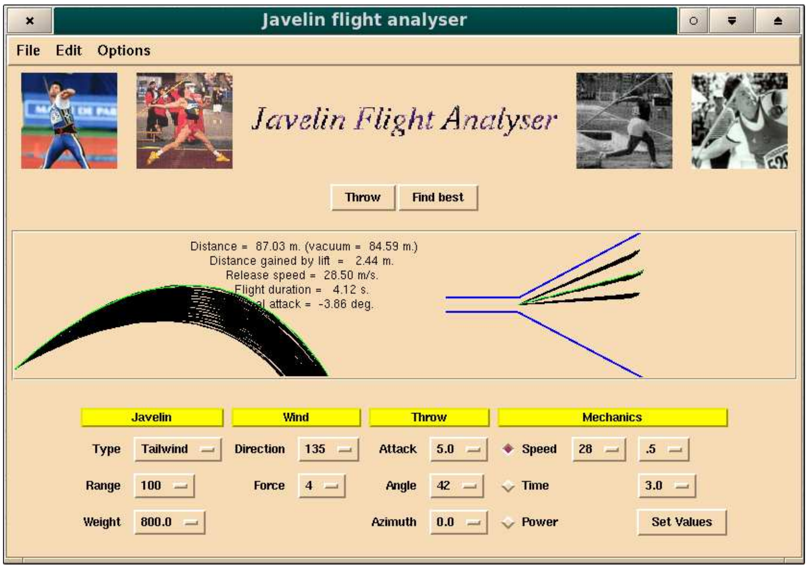

Figure 7: A screenshot of the front page of the Javelin Flight Analyser after performing an optimal search for a wind from a bearing of 135 degrees taking 0 degrees as straight down the centre of the fan, (this would correspond to a wind from behind the right shoulder of the thrower).

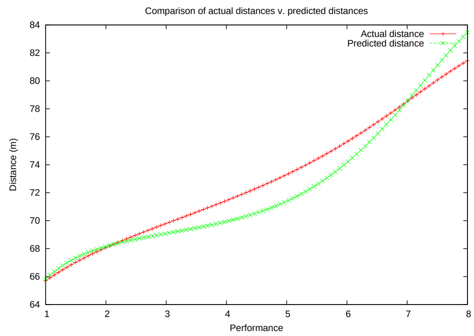

Figure 8: The result of predicting distances using the simple parabolic equations of section 3.1.

Figure 8 shows the distances using the basic parabolic development of section 3.1. As can be seen, this works quite well for some performances but not for others. It should be noted that sometimes over-prediction occurs and sometimes under-prediction. Any departures from this ideal parabolic solution *must* be because of the effects of air resistance. Figure 9 shows the predicted distances using the sophisticated model described in section 3.2. This is comparable only in accuracy to the simple parabolic model although the fit is different. However, the real value of the sophisticated model can be seen when simple assumptions are made about the presence of a typically capricious wind varying between Beaufort force 0 and 2, (a gentle wind) and varying also in direction as is typical in stadia. This alone is sufficient to allow a fitting accuracy as good as that shown in Figure 10 to be done.

This can also be done with the data quoted by [1] taken at the 1991 World Student Games. The equivalent wind-fitted data is shown in 11. In this case, the assumption of a head wind of around force 2 was necessary to produce this fit for the Woman's event and the assumption of a slight tail wind of around force 1 was necessary for the Men's event. It is intriguing to note that these assumptions were consistent for all performers and rather different for the two events, which were held at different times.

Note that this does not of course prove anything. However, no wind data is present for any of the quoted experiments to verify this more sophisticated calibration. It suffices to show here therefore that a gentle wind can account for all of the errors in the sophisticated model *without any other change to modelling parameters being necessary*. Moreover in each of the three cases, it led to a typical wind scenario which was then consistent with all performers which gives some qualitative support for the approach used here. True confidence in this approach can only however spring from measurement and experiments of the nature of [1] will have to be repeated with careful measurement of the prevailing wind at the time of each performance to validate the model fully.

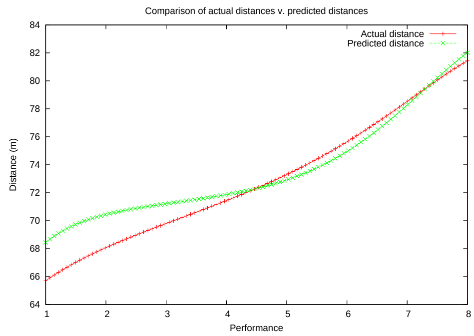

Figure 9: The result of predicting distances using the sophisticated coupled non-linear differential equations of section 3.2.

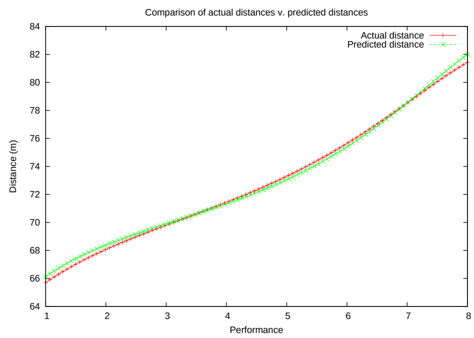

Figure 10: The result of predicting distances using the sophisticated coupled non-linear differential equations of section 3.2 including the effects of a gentle wind variable in direction.

The availability of a sophisticated model to predict the length of a javelin throw opens up the possibility of modelling a scenario in many different ways to find the optimum set of throwing parameters for a given prevailing wind and set of biomechanical restrictions. The computer model is already capable of doing this and has already produced some interesting results. For example, in the case of Figure 7, if the thrower is capable of matching the specified parameters, (which can easily be restricted to match his or her biomechanical characteristics), the best throw is down the middle in this case with the attack angle shown. The wide range of inferior solutions should also be noted here. The nature of these will be explored in more detail when suitable calibration data for prevailing wind becomes available.

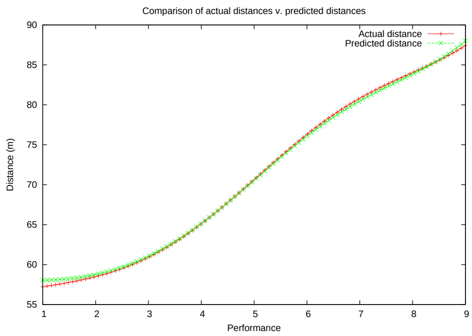

Figure 11: Wind fitted data for the sophisticated model of 3.2 for the 1991 World Student's Games held in Sheffield, UK. Both Men and Women's events are shown here although they were held separately and slightly different prevailing wind scenarios were used to achieve this quality of fit.

7 Conclusions

A sophisticated model of javelin flight including the effects of prevailing wind conditions has been presented along with a freely downloadable Windows program which implements it. The model has been calibrated against two sets of published data and it produces good qualitative predictions for javelin flight in different wind conditions and is in good agreement with the small number of observations available in the literature. The model has proven to be capable of fitting the data to better than 1% but data on prevailing wind conditions will be necessary to validate this level of accuracy.

The model also shows that it is not always obvious how to throw a javelin best in different conditions and there are some surprises for throwers. The model also supports recent insight into how different models of javelin fly, in particular some of the long-standing misunderstandings of Tailwind javelins. Finally it is very clear that for wind-sensitive events like the javelin and discus, it is very important to understand the physics of the flight and the effects of the prevailing wind conditions on performance.

8 Acknowledgements

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References

[1] R.J. Best, R.M. Bartlett, and C.J. Morriss. A three-dimensional analysis of javelin throwing techniques. *Journal of Sports Science*, 11(4):315–328, August 1993.

[2] R.J. Best, R.M. Bartlett, and R.A. Sawyer. Optimal javelin release. *Journal of Applied Biomechanics*, 11:371–394, 1995.

[3] N. Curle and H.J. Davies. *Modern Fluid Dynamics: Vol 1 incompressible flow.* van Nostrand, 1968.

[4] L. Hatton. Javelin flight analyser. *http://www.leshatton.org/javelin_2005.html*, May 2005.

[5] K. Bartonietz K.D. Maier, V. Wank and R. Blickhan. Neural network based models of javelin flight: prediction of flight distances and optimal release parameters. *Sports Engineering*, 3:57–63, 2000.

[6] C. Morriss and R. Bartlett. Biomechanical factors critical for performance in the men's javelin throw. *Sports Medicine*, 21(6):438–446, June 1996.

[7] C.J. Morriss, R.M. Bartlett, and N. Fowler. Biomechanical analysis of the men's javelin throw at the 1995 world championships in athletics. *Thrower*, 78:18–31, July 1998.

[8] B. Parkes. Notes on head and tailwind javelins by Dick Held dated 7 june, 2004 and 16 july 2005. *Personal Communication*, May 2005.

[9] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes in C.* Cambridge University Press, 2nd edition, 1999. ISBN 0-521-43108-5.

[10] Milton van Dyke. *Perturbation methods in fluid mechanics.* Academic Press, 1970. ISBN 12-713050-0.